Introduction to the Laboratory: Measurements, Uncertainty and Significant Figures

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Purpose of the Exercise:

- Learn how to read any graduated scale correctly
- To be able to read and record measured values correctly.
- To master the concept of significant figures as the meaningful digits in a measurement
- To be able to determine the correct number of significant figures when reporting results of calculations.

Background Required:
An open mind and a willingness to master new concepts in making measurements and performing calculations. You should also be comfortable using scientific notation.

Background Information:
Welcome to the laboratory portion of this course! In our laboratory experiments throughout the term, we will be manipulating matter and recording measurements of mass, volume, length, temperature and even the barometric pressure using common laboratory equipment. One of our goals is to learn how to make and record these measurements as precisely and as accurately as possible, so we can achieve more accurate results AND understand the limitations of our data. You have been making measurements since early childhood, using the ruler in your school supplies, the scale in the kitchen or bathroom, various thermometers, and the speedometer in your vehicle. If the read-out was digital, you may have noted the integer digits and mentally rounded off any decimal places. For devices with graduated marks like a ruler, you may have just rounded the measurement to the nearest marking on the ruler. For the majority of everyday measurements this approach is fine; our cake in the oven is unlikely to be affected by a temperature difference of 0.2 degrees and our bathroom scale is unlikely to be able to reliably detect a difference of 0.5 pounds.

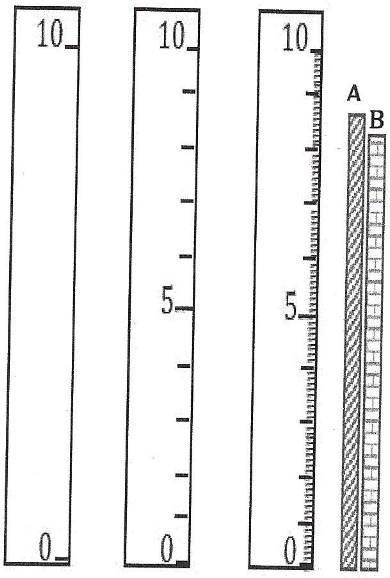
In the laboratory, the rules change. We need to understand both the precision and the limitations of our measuring devices. We want to be able to measure small differences in mass, volume or physical and chemical properties and to compare our measurements with known reference values. We want to perform reactions with carefully measured quantities of reactants to control how our reactions proceed. Most importantly, advances in science are often made by detecting very small differences in experimental outcomes; these differences can be completely obscured if the measurements are not made and reported correctly.

Uncertainty in Measurements
One of the most important concepts to understand is that every measurement has some uncertainty associated with it. The amount of uncertainty is determined by the measurement device. A user may make errors in reading a device, but the device itself has an inherent limit to the smallest difference it can detect in a measured value. Let's explore this concept using a simple linear scale, like a ruler.

On the next page is a figure with 3 different scales, each with different division marks. Scale 1 has marks only at 0 and 10 units; the difference between the closest marks is 10 units. Scale 2 has marks every 1 unit, while on Scale 3 the closest markings are 0.1 unit apart.

The correct way to read any graduated scale such as these is to read to the nearest mark and estimate one (*and only one*) more decimal place between the closest marks. This estimated digit represents the last digit that would provide meaningful information using that scale and has some uncertainty associated with it. Estimation to another decimal place is not possible with any certainty, so **digits after the last estimated digit are meaningless and should not be recorded**.

In order to see the impact of measurement device uncertainty in science, we will try to determine the difference in the length of the two patterned bars A and B. Using each scale, measure the length of Bar A and B to the nearest mark and estimate the next digit. Record your measurements in the table below and calculate the difference in the length of the bars. Note that the measurement for Scale 1 will only have one digit, the estimated digit.

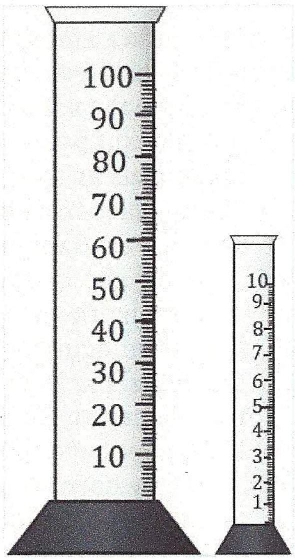


Bar Letter	**Scale 1**	**Scale 2**	**Scale 3**
A			
B			
Difference in length			

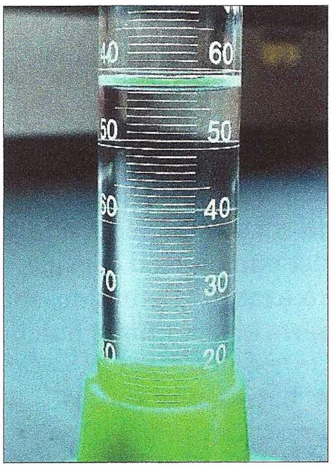
The difference in the length of the bars using Scale 1 should be either 0 or 1 unit, depending on your estimate of each bar's length. Visually, it is easy to see that there is a difference in the lengths and that the difference is less than one unit, but the uncertainty inherent in trying to make a measurement using Scale 1 does not allow us to accurately measure the difference. Using Scale 2, the bar lengths can be read to 1 unit and estimated to 0.1 unit, so we record our lengths using 2 digits. After subtracting the length of Bar B from the length of Bar A, we now can report that the difference in the bars to a tenth of a unit. Scale 3 allows us to read the length to 0.1 unit and to estimate one more digit. When we try to estimate the between the smallest marks on Scale 3, we experience the limitation of the device and our human eyes, so our estimation becomes less certain. We may only be able to estimate the last digit as **on the mark** (8.7$\underline{0}$ units) or **between the marks** (8.7$\underline{5}$ units). In either case, we are much more certain of our measurement using Scale 3 than Scale 1 and we report the measurement with 3 digits. When we make a measurement, the digits we can read from the markings on the scale plus the *one* estimated digit are called the **significant figures or significant digits**. The **more significant figures** we have in a measurement, **the more precise the measurement** is. In addition, having more significant digits in a measurement allows us to detect much smaller differences in the quantity we are measuring.

In the lab, we will be measuring volumes of liquids using graduated cylinders. Let's practice the above approach to making measurements with a graduated scale, paying close attention to the number of digits we will record with different size graduated cylinders. When you look at any graduated scale, you should note the numerically marked units of the scale (ml, cm, g, etc.) and the difference in the unit represented by the smallest marks.

The figure shows typical scale markings for a 10 mL and a 100 mL graduated cylinder.
Examine the 100 mL cylinder: What is the difference in volume for the marks with numbers? ________
What volume difference do the smallest marks show? _________
How many decimal places should we record when making volume measurements with this cylinder? ________
Use the same process and determine how many decimal places volume readings for the 10 mL cylinder should have: __________.
You should now be able to confidently read any graduated scale and record a measurement with the correct significant figures. Try reading the volumes for the graduated cylinders below. **Read from the <u>bottom</u> of the curve** of the liquid (**called the meniscus**) to the smallest mark and estimate one more decimal place between the marks.



Graduated Cylinders



Volume __________mL

Volume _________mL

Top Loading Balance

Digital Displays: Laboratory devices, such as the top-loading balances we use to measure mass, often provide digital displays of measured values. Even with a digital display, the last digit displayed is assumed to have some uncertainty associated with it. All the other digits represent measured values. When using a balance or any measuring device with a digital display, **all digits** that are displayed are considered **significant figures** and **should be recorded**.
How many significant figures are there in the balance reading shown? ______________________
What should you write for the mass of the crucible on the balance? __________________________

Recognizing Significant Figures
In addition to being able to record measurements with the correct significant figures, we also need to be able to recognize the number of significant digits in a given number, whether the number is a measurement, a constant or a conversion factor. There are simple rules for determining which digits in a given number are significant and which are only placeholders.

1. All **non-zero digits** in a given number are **significant**.
2. All zeros between non-zero digits (called **interior zeros**) are **significant**. These **interior** zeros have been measured and thus represent meaningful values.

3. All **leading zeros**, before or after a decimal point, are **not significant.** **Leading zeros** will disappear when we write a number using scientific notation, so they are not significant. For example, the value 0.00567 becomes 5.67 x 10^{-3} when written using scientific notation and has 3 significant figures.
4. **Trailing zeros after a decimal point are significant.** Trailing zeros are only included in a value when values have been measured or estimated for those decimal places, so they are significant figures. For example, a measurement given as 8.30 cm means a length has been measured to 8.3 cm using the marks on the ruler and the final digit has been estimated as 0 (as being on the smallest mark and not between marks). This final 0 is significant and included in our number of significant figures.
5. **Trailing zeros in values that do not have a decimal point are ambiguous,** that is, we cannot be sure if they represent measured values or are place holders. For example, a value written as 1500 may have been measured to the "ones" place, so all the digits are significant, or it may have only been measured to the "hundreds" place, so only the 1 and 5 are significant figures. To avoid this confusion, we write numbers such as 1500 using scientific notation so we can show which of the zeros are significant. If all the digits have been measured, we would write 1.500 x 10^3. If only the first two digits were measured, we would write 1.5 x 10^3.
6. **Exact numbers and significant figures:** Many numbers we encounter are exact numbers and the significant figure rules do not apply. Numbers representing quantities that have been counted, such as "5 students" are exact numbers and are considered to have an infinite number of significant figures. Other examples of exact numbers include metric conversion factors (1 m = 100 cm) or any defined mathematic expression with an integer ratio such as the equation for the circumference of a circle (circumference = $2\pi r$). The "2" is an exact number and considered to have infinite significant figures. Sometimes conversion factors are exact by definition, such as 1 inch = exactly 2.54 cm. In this case, the "2.54" is an exact number with an infinite number of significant figures implied.

Calculations with Significant Figures

Each measurement we make in lab will have a specific number of significant figures, based on the measuring device. We will often be using our measurements to calculate other values and we need to know how to keep track of significant figures in calculations in order to write the results of our calculation with the correct number of significant figures. We must be aware that most **calculators do not track significant figures**. Your calculators will not display trailing zeros after a decimal point, even if those zeros are significant. Your calculators will, however, display many calculated digits that are **not** significant. You must use your knowledge of the significant figures of each value used in a calculation to be able to report the result with the correct significant figures. Luckily, the rules for determining the number of significant figures to include in a result are fairly simple. The general principle is that a calculated answer cannot be more precise than the least precise measurement from which it was calculated

1. Multiplication and division: When multiplying or dividing values, the final result should be written with the **smallest number of significant figures** from any of the values used. For example: the result of this calculation 2.50 x 3.2 /0.422 = 18.957345, etc. should be written with either 2 significant figures as 19 or the answer can be shown with additional digits as long as the last significant figure is underlined (18.957) to indicate the result only has 2 significant figures. When performing calculations with multiple steps, it is better to use the underlining approach to track the significant figures and keep a few extra digits at each step. Only the final answer is then rounded to the correct number of significant figures.
2. Addition and subtraction: When adding or subtracting values, the **number of decimal places** for each value **controls the number of digits that are considered significant.** The final answer

can have no more decimal places than the value with the **least number of decimal places**. In the examples below, we find the number of significant figures for the answer by first determining how many decimal places the answer should have. We can use the approach of underlining the last significant figure here as well.

Addition:		Subtraction	
0.85			
5.257		10.85	
+6.0		- 2.033	
12.$\underline{1}$07	*1 decimal place* *3 significant figures*	8.8$\underline{1}$7	*2 decimal places* → 8.82 *3 significant figures*

Notice that the number of significant figures in the final answer can be different than any of the data values, which is not the case with multiplication and division.

3. Mixed operations: If we have a calculation involving both addition subtraction and multiplication/division, we use the normal order of operations and determine the number of significant figures for each step in the calculation. The normal order is:
    a. operations in parentheses
    b. exponents
    c. multiplication/division
    d. addition/subtraction

   For the type of calculations we will do, the easiest approach is to put any addition/subtraction calculations in parentheses, do that calculation first and determine the correct number of significant figures for the result. The significant figures in this result are then considered when performing the next calculation and determining the number of significant figures in the final answer.

   Here is an example using the formula for determining the percent error in an experimental result. The difference between the experimental value and the "true" value is calculated, then the result is divided by the "true" value and multiplied by 100% (note: 100% is an *exact number*). In this calculation, the subtraction operation is put in parentheses and performed first. The difference is calculated and evaluated for decimal places and then significant figures. Then the next operation is performed and the significant figures in the final answer are determined.

$$\text{Percent error} = \frac{(\text{experimental value} - \text{true value})}{\text{true value}} \times 100\%$$

2 decimal places; 1 significant figure

$$\text{Percent error} = \frac{(6.31-6.332)}{6.332} \times 100\% = \frac{(0.0\underline{2}2)}{6.332} \times 100\% = 0.\underline{3}4744 \Rightarrow \mathbf{0.3\%} \quad \mathbf{1\ SF}$$

   The result of the subtraction operation can only have 2 decimal places, based on our experimental value, which results in only one significant figure (SF). Our final answer after the division step can only have one significant figure.

Rounding final answers: To round answers to the correct number of significant figures, we look at the digit to the right of the last significant figure. If that digit is 4 or less, we do not change the value of the last significant digit and we drop any digits further to the right. If the digit to the right of the last significant figure is 5 or more, we increase the value of the last significant digit by one and drop all other digits to the right. If you are unsure of how to round a very large or small number, write the value in scientific notation and use the above rules.

Practice Exercises:
How many significant figures are there in each of the following values?

Number	Significant figures
1.357	
0.00230	
150,001	
6.305×10^3	

Number	Significant figures
.000300	
6592	
2.003×10^{-5}	
18.0080	

Perform the calculations and round the final answer to the correct significant figures.

Calculation	Answer given by calculator	Answer in scientific notation and rounded to correct significant figures
1.35 + 0.253 + 217 =		
2652.1 – 85.7 + 2.19 =		
107.8 x 0.6930 x 132 =		
12.2 x 8.75 /0.0034 =		
(3.82 – 3.005)/ 3.005 =		
$(1.23 \times 10^2) \times (5.467 \times 10^{-4}) =$		

Calculations with experimental data

1. The density of an object is the ratio of its mass to its volume or **Density** $= \frac{\text{mass (g)}}{\text{volume (mL)}}$.
   The mass and the volume of a plastic ball were measured in an experiment and the following values were obtained: **Mass: 14.530 g Volume: 30.6 mL**
   Calculate the density of the plastic ball (use correct significant figures and units in answer).

   Density = __________________

2. The true value of the density of the plastic ball is 0.468 g/mL. Calculate the percent error in using the equation below. Show your set-up and answer with correct significant figures.

$$\text{Percent error} = \frac{\text{(experimental value} - \text{true value)}}{\text{true value}} \times 100\%$$

Percent error = ____________